Preprint typeset in JHEP style - PAPER VERSION

hep-ph/0703086
IPPP/07/05
DCPT/07/10

Naturalised Supersymmetric Grand Unification

Steven A. Abel, Joerg Jaeckel and Valentin V. Khoze

Institute for Particle Physics Phenomenology and Centre for Particle Theory, University of Durham, Durham, DH1 3LE, UK
s.a.abel@durham.ac.uk, joerg.jaeckel@durham.ac.uk, valya.khoze@durham.ac.uk

Abstract: We construct a simple model of an $SU(5)$ GUT with gauge mediated supersymmetry breaking from a metastable vacuum of a hidden sector. All mass parameters and hierarchies of our model are generated dynamically from retrofitting. This includes the μ-parameter and the GUT scale. However, as typical for $SU(5)$ GUTs, proton longevity remains a problem.

1. Introduction

The mechanism whereby supersymmetry is broken in Nature is once again the subject of intense scrutiny. Of particular importance has been the realization by Intriligator, Seiberg and Shih (ISS) that (for appropriate choices of flavours and colours) the simplest SQCD models have SUSY breaking metastable minima [1]. Such models are phenomenologically acceptable provided the decay time from the metastable to the supersymmetric vacuum is sufficiently long. Furthermore, it was argued that the early Universe is naturally driven to such metastable minima and remains trapped there [2–6]. Metastability allows for the presence of supersymmetric 'true' vacua in the theory and thereby allows one to evade several stringent constraints on supersymmetry breaking. These include the Nelson–Seiberg theorem [7] which requires an R-symmetry leading to unwelcome phenomenological consequences, such as vanishing gaugino masses or the presence of $R-$axions.

Accepting metastable SUSY breaking minima [1] (for earlier work see [8–10]) leads to a far broader class of SUSY breaking models which is very appealing. This fact is exploited in the *retrofitting* approach of [11, 12] which – in the light of the ISS model – generalizes and greatly improves upon earlier models of metastable SUSY breaking. The approach begins with a model that has an exact R-symmetry, and breaks it with terms that are generated dynamically and are thus small. The models are metastable, but the fact that R-symmetry is still approximately conserved is enough to ensure that the global SUSY preserving minima are far away in field space and hence the SUSY breaking minima are long lived. There have since been a number of discussions of how such metastable SUSY breaking might be mediated to the Standard Model, including direct mediation [1, 13, 14], breaking within the visible sector [15] and gauge mediation [11, 12, 16–18].

Our purpose in this paper is to examine the consequences of these developments for Grand Unification. In particular, the retrofitting programme seeks to explain all mass scales dynamically by the confinement of hidden gauge sectors. As well as the SUSY breaking scale itself, one would naturally like to obtain explanations for other dimensionful parameters such as the μ-term of the MSSM (as in e.g. [12]). Grand Unified Theories (GUTs) are of course full of dimensionful parameters: the GUT scale; the SUSY breaking scale; the μ-term of the effective low energy theory. In the simplest $SU(5)$ GUTs, the latter is especially bothersome, requiring a fine-tuning between mass parameters to one part in 10^{14}, the so-called doublet-triplet mass-splitting problem (for a review see [19]). One is led to ask whether GUTs can be made more natural in the light of metastability: is it possible to retrofit a GUT model with broken supersymmetry entirely, so that no dimensionful parameters have to be chosen by hand?

Basing our analysis on the simplest examples of gauge mediation developed in refs. [16, 17] we will argue that it is. The former paper outlined a simple model of gauge mediation, whereas the latter showed how it can be retrofitted, with all mass terms being generated dynamically. However, neither considered the coupling to, or parameters of the MSSM, such as for example the μ-parameter. Our objective in the present work will be to completely retrofit this parameter as well as the other parameters required for GUT and SUSY breaking itself: in other words to construct a theory whose

GUT breaking, SUSY breaking, messenger scale and μ-term are all generated by the dynamics. We will be able to generate and explain within this approach the three key scales of the visible sector: the GUT scale $\sim 10^{16}$ GeV, the electro-weak and the supersymmetry breaking scales, both $\sim 10^{2}$ GeV. In particular, our model predicts a relation between the GUT and the elctroweak scale,

$$M_{GUT} \approx (16\pi^2 \mu_{MSSM} M_p^5)^{1/6} \ . \tag{1.1}$$

For this preliminary study we will be considering the simplest case which is a minimal $SU(5)$ GUT (for a review see [20]). These models are known to conflict with bounds on the decay rate of the proton because of large dimension-5 operators mediated by Higgs triplets. Indeed in the model we present here, the Higgs triplets are lighter than usual (although still relatively close to the GUT scale) so that the proton decay rate is significantly worse. Nevertheless the model is an encouraging first step on the road to a fully consistent retrofitted GUT. We discuss in a later subsection how the model or similar GUT models may be developed in order to make it more realistic.

2. The model

We want to construct a simple and predictive model which combines and inter-relates the ideas of supersymmetric Grand Unification [21], supersymmetry breaking by a metastable vacuum [1], and naturalness achieved through retrofitting [12].

Following the general set-up of [17] we consider a model made up of three sectors.

1. The first is the *R-sector* whose main rôle is to dynamically generate all mass-parameters in the effective Lagrangian of the full model. This is achieved via a version of the retrofitting approach of [11, 12] which will be reviewed shortly. In our model this sector is described by a strongly coupled confining SQCD theory with the dynamical scale Λ_R. In the full theory Λ_R triggers the dynamical generation of masses as in [17]. In addition, in our model the $N_f \times N_f$ meson superfield $\tilde{Q}_R Q_R$ of the R-sector will play the rôle of the adjoint Higgs of the GUT sector.

2. The second sector is responsible for supersymmetry breaking. It is described by the SQCD in a free magnetic phase, known as the ISS model [1]. This model contains a long-lived metastable vacuum which breaks supersymmetry, and will be referred to as the metastable susy-breaking, or *MSB-sector*.

3. The visible sector is the $SU(5)$ susy *GUT-sector*. The $SU(5)$ gauge group arises from gauging the flavour $SU(N_f = 5)$ symmetry of the R-sector, and the adjoint Higgs field Φ_{GUT} is identified with the traceless part of the R-sector mesons $\tilde{Q}_R Q_R/\Lambda_R$. The GUT-sector is coupled to the MSB-sector via messenger fields f and $\tilde{f}$ which are in the fundamental and the anti-fundamental of the $SU(5)$ gauge group. Hence supersymmetry breaking is mediated to the GUT theory via gauge mediation.

In what follows we will see that this model delivers a supersymmetric Grand Unified Theory with calculable soft susy-breaking terms (arising from interactions with the MSB-sector). The model is fully natural and all the mass-scales of the theory are generated in terms of appropriate combinations of the two dynamical scales Λ_R, Λ_{MSB} and the Planck scale M_p. In particular, by choosing Λ_R and Λ_{MSB} our model naturally generates the desired values of the electro-weak, supersymmetry breaking $\sim 10^2$ GeV, and the GUT scale $\sim 10^{16}$ GeV.

2.1 Interactions between the sectors

Now we proceed to specify the interactions *between* the three sectors of the model. These are introduced through the superpotentials $\mathcal{W}_1$, $\mathcal{W}_2$ and $\mathcal{W}_3$ with one property in common: they couple bilinears from one sector to a bilinear from another and as such are represented by lowest-dimensional non-renormalizable operators suppressed by M_p. For simplicity of presentation, in equations (2.1), (2.6), (2.10) we will include only the interactions which are necessary for our model. Other interactions will be discussed in the Appendix. The superpotential $\mathcal{W}_1$ is responsible for the retrofitting [12, 17] and couples the singlet bilinear made of the gauge-strength superfield W_R of the confining R-sector to the singlet bilinears of the MSB- and the GUT-sectors:

$$\mathcal{W}_1 = \mathrm{tr}(W_R^2)\left(\frac{1}{g_R^2} + \frac{\mathrm{const}}{16\pi^2 M_p^2}\,\mathrm{tr}(\tilde{Q}_{MSB}Q_{MSB}) + \frac{\mathrm{const}}{16\pi^2 M_p^2}\,\mathrm{tr}(\tilde{f}f) + \frac{\mathrm{const}}{16\pi^2 M_p^2}\,\mathrm{tr}(\tilde{H}H)\right) , \qquad (2.1)$$

where $\tilde{Q}_{MSB}$, Q_{MSB} are the (anti)-fundamental quark superfields of the MSB sector, $\tilde{f}$, f and $\tilde{H}$, H are the messengers and the Higgs fields transforming in the (anti)-fundamental of the $SU(5)$ GUT. The const's on the right hand side of (2.1) are of order one, and factors of $1/16\pi^2$ indicate that these contributions come from loop effects in the underlying theory at the scale M_p. These are the leading-order higher-dimensional operators which involve interactions between WW and the matter-field bilinear gauge singlets. Operators of even higher dimension will be suppressed by extra powers of the Planck mass M_p and will not be relevant for our analysis.

The R-sector is described by a non-Abelian gauge theory. We will take it to be an SQCD theory with the gauge group $SU(N_c)$ and N_f flavours of quarks $\tilde{Q}_R$, Q_R with $N_f < N_c - 1$. The quark fields $\tilde{Q}_R$, Q_R develop (large) VEVs which break the gauge group to $SU(N_c - N_f)$. The resulting 'low-energy' theory of the R-sector is the pure $SU(N_c - N_f)$ SYM with the dynamical scale Λ_R (plus colour-singlet meson fields $\tilde{Q}_R Q_R$). The SYM theory is strongly-coupled at the scale Λ_R and develops a gaugino condensate,

$$\langle W_R^2 \rangle = \langle \lambda_R^2 \rangle = \Lambda_R^3 . \qquad (2.2)$$

This effect in the superpotential (2.1) generates masses $m_{Q_{MSB}}$, m_f and m_H of the order $\sim \Lambda_R^3/M_p^2$ for the appropriate chiral matter fields. This mass generation is the retrofitting mechanism of [12] as explored recently in [17] in the ISS model building context. A novel feature of our model compared to [17] is the fact that in our context not only the MSB-quarks and the messengers, but also the GUT Higgs fields H and $\tilde{H}$ get a retrofitted mass m_H which gives rise to the μ_{MSSM} parameter of the

Standard Model,

$$\mu_{MSSM} \equiv m_H \sim \frac{1}{16\pi^2}\frac{\Lambda_R^3}{M_p^2} \ . \tag{2.3}$$

The generation of the quark masses $m_{Q_{MSB}} \sim \Lambda_R^3/M_p^2$ is a key ingredient for the metastable susy breaking [1] in the MSB sector. The relevant scale is [17]:

$$\mu_{MSB}^2 \equiv \Lambda_{MSB} m_{Q_{MSB}} \sim \frac{1}{16\pi^2}\frac{\Lambda_{MSB}\Lambda_R^3}{M_p^2} \ . \tag{2.4}$$

In the context of our model, the generation of μ_{MSSM} in (2.3) and μ_{MSB} in (2.4) are the only relevant effects of the retrofitted superpotential (2.1). The value of $\Lambda_R \gtrsim 10^{14}$ GeV is then chosen, so as to give

$$\mu_{MSSM} \sim \frac{1}{16\pi^2}\frac{\Lambda_R^3}{M_p^2} \gtrsim 10^2 \div 10^3 \,\text{GeV}, \ . \tag{2.5}$$

as required for electro-weak symmetry breaking.

Although the messenger fields $f, \tilde{f}$ also get a contribution to their masses from $\mathcal{W}_1$, the dominant contribution to m_f comes from a second class of interactions between gauge singlets from different sectors. These couple the messenger fields of the GUT sector and the quark bilinears from the hidden sectors;

$$\mathcal{W}_2 = \frac{\text{const}}{M_p}\,\text{tr}(\tilde{f}f)\,\text{tr}(\tilde{Q}_{MSB}Q_{MSB}) + \frac{\text{const}}{M_p}\,(\tilde{f}f)\,(\tilde{Q}_R Q_R) \ . \tag{2.6}$$

These terms are ultimately responsible for the mediation of susy-breaking from the MSB-sector to the GUT-sector, and specifically for the generation of Majorana gaugino masses. The traces in (2.6) are over gauge and flavour indices of each sector. Furthermore, as mentioned earlier, the flavour symmetry $SU(N_f = 5)$ of the R-sector is gauged, and this makes the R-meson field $\tilde{Q}_R Q_R$ an adjoint plus a singlet under the GUT $SU(5)$ gauge group,

$$\tilde{Q}_R^i Q_R^j = \Lambda_R \Phi_{GUT}^{ij} \ , \qquad i,j = 1\ldots N_f = 5 \ . \tag{2.7}$$

We will show in the next subsection that the VEV for $\tilde{Q}_R^i Q_R^j$ is generated dynamically in the R-sector of our theory and is of the form

$$\frac{1}{\Lambda_R}\langle \tilde{Q}_R^i Q_R^j \rangle = M_{GUT}\,\text{diag}(+1,+1,+1,-1,-1) \ , \ . \tag{2.8}$$

The mass term for the messengers arises from the last term[1] in (2.6). Using (2.8) we find

$$m_f \sim \frac{\Lambda_R}{M_p} M_{GUT} \ . \tag{2.9}$$

The third class of interactions couples the Higgs (anti)-fundamental fields of the GUT sector to the adjoint (plus a singlet) Higgs which arises from mesons of the R-sector. It has the form,

$$\mathcal{W}_3 = \frac{\text{const}}{M_p} H \cdot \Big(\text{tr}(\tilde{Q}_R Q_R) + \tilde{Q}_R Q_R\Big) \cdot \tilde{H} \ . \tag{2.10}$$

[1]The structure of the last term in (2.6) is a short-hand for a generic interaction, consistent with an unbroken $SU(5)$, $f \cdot (c_1 \text{tr}(\tilde{Q}_R Q_R) + c_2 \tilde{Q}_R Q_R) \cdot \tilde{f}$, where $c_{1,2}$ are constants of order 1.

These two terms are included to raise the mass of the Higgs triplet fields and do not give any additional mass to the doublets. In order for this to be the case we require the couplings to be precisely equal as shown. The doublet-triplet splitting will be discussed in more detail below.

2.2 R-sector and the generation of the GUT scale

In our approach all mass-parameters should be generated dynamically. An important point then is to explain how the GUT scale $M_{GUT} \sim 10^{16}$ GeV is generated alongside the much lower μ_{MSSM} scale in (2.5). In this sub-section we will show that this hierarchy of scales is naturally explained by the dynamics of the R-sector of our model.

As already mentioned, the R-sector is given by an SQCD with $N_c > N_f + 1$, with the number of flavours being set to $N_f = 5$. The quarks are exactly massless since in the general set-up which we follow no tree-level masses can be put in by hand. As is well-known, there is a nonperturbative Affleck-Dine-Seiberg superpotential [22] in this theory which leads to run-away vacua and renders the theory inconsistent, unless there is a mechanism to prevent the run-away and stabilize the vacua. Without loss of generality and naturalness, this is easily achieved by adding a leading-order higher-dimensional operator to the Lagrangian, so that the total superpotential for the meson fields of the R-sector is,

$$\mathcal{W}_R = (N_c - N_f)\left(\frac{\Lambda_{SQCD}^{3N_c-N_f}}{\det_{N_f}(\tilde{Q}_R Q_R)}\right)^{\frac{1}{N_c-N_f}} + \frac{1}{2M_p}\mathrm{tr}(\tilde{Q}_R Q_R)^2 \ . \tag{2.11}$$

The dynamical scale Λ_{SQCD} appearing in the Affleck-Dine-Seiberg superpotential above, is the scale of the full SQCD theory of the R-sector, and should be distinguished from the dynamical scale Λ_R of the 'low-energy' $SU(N_c - N_f)$ pure SYM. The relation between Λ_{SQCD} and Λ_R will be determined below.

In terms of the meson field $M_{ij} = \tilde{Q}^i_R Q^j_R$ the F-flatness condition on (2.11) gives an equation for diagonal components (without loss of generality we work in the basis where $\langle M_{ij}\rangle$ is diagonal),

$$\langle M_{ii}\rangle^2 = \langle M\rangle^2 \left(\frac{\langle M\rangle^{N_f}}{\det_{N_f} M}\right)^{\frac{1}{N_c-N_f}} \ , \tag{2.12}$$

which holds for each value of $i = 1, \ldots, N_f = 5$, and where

$$\langle M\rangle = \Lambda_{SQCD}^2 \left(\frac{M_p}{\Lambda_{SQCD}}\right)^{\frac{N_c-N_f}{2N_c-N_f}} \ . \tag{2.13}$$

Since the right hand side of (2.12) does not depend on i it follows that all the values of $\langle M_{ii}\rangle^2$ must be equal to each other. However this does not necessarily imply that the VEVs of the meson field itself are all the same. For $N_f = 5$ there are three inequivalent discrete solutions of (2.12), the first one is

$$\langle M_{ij}\rangle = \langle M\rangle\,\mathrm{diag}(+1,+1,+1,+1,+1) \quad => \quad SU(5) \ , \tag{2.14}$$

the second solution breaks $SU(5)$ down to $SU(4)$,

$$\langle M_{ij}\rangle = \langle M\rangle\,\mathrm{diag}(+1,-1,-1,-1,-1) \quad => \quad SU(4) \ , \tag{2.15}$$

while the third solution is precisely what we require, it corresponds to a spontaneous breakdown of $SU(5) \to SU(3) \times SU(2) \times U(1)$,

$$\langle M_{ij} \rangle \; = \; \langle M \rangle \, \mathrm{diag}(+1, +1, +1, -1, -1) \quad => \quad SU(3) \times SU(2) \times U(1) \; . \tag{2.16}$$

The vacuum expectation value of the meson field in (2.13) should now be expressed in terms of the dynamical scale Λ_R for the effective pure SYM $SU(N_c - N_f)$ theory. This is easily achieved by first recalling that Λ_R is related to the gaugino condensate of the SYM theory via

$$\Lambda_R^3 \; = \; \langle \lambda_R^2 \rangle \; , \tag{2.17}$$

which in turn is given by the vacuum value of the superpotential (2.11),

$$\langle W_R \; \sim \; \frac{1}{M_p} \langle M \rangle^2 \; . \tag{2.18}$$

This gives

$$\langle M \rangle \sim \sqrt{\Lambda_R^3 M_p} \; , \tag{2.19}$$

in terms of Λ_R, which is precisely what we are after.

Finally, we need to define a canonically normalised meson field Φ_{GUT} in terms of the dimension-two meson field we were using so far. There are essentially two dimensionful parameters, $\sqrt{\langle M \rangle}$ and Λ_R in the QCD theory of the R-sector, which obey $\sqrt{\langle M \rangle} \gg \Lambda_R$. The first parameter sets the scale where the full $SU(N_c)$ is broken down to $SU(N_c - N_f)$, and the second parameter, is the confinement scale of the $SU(N_c - N_f)$ SYM. It can be argued that the dimension-one field which is normalised canonically in the Kahler potential, is the one which uses the lowest scale, i.e. Λ_R,

$$\Phi_{GUT}^{ij} \; = \; \frac{1}{\Lambda_R} \, \tilde{Q}_R^i Q_R^j \; . \tag{2.20}$$

Fields normalised with $(\sqrt{\langle M \rangle})^{-1}$ would give subdominant contributions to the Kahler potential. Also, the normalisation (2.20) is consistent with the normalisation of the meson field in the ISS theory [1].

In total we have

$$\langle \Phi_{GUT}^{ij} \rangle \; = \; M_{GUT} \, \mathrm{diag}(+1, +1, +1, -1, -1) \; , \quad \text{where} \quad M_{GUT} \; \sim \; \sqrt{\Lambda_R M_p} \; . \tag{2.21}$$

Taking the same value of $\Lambda_R \; \gtrsim \; 10^{14}$ GeV as required for electroweak symmetry breaking in (2.5) we find

$$M_{GUT} \; \sim \; 10^{16.5} \, \mathrm{GeV} \; . \tag{2.22}$$

Or, in other words, by eliminating Λ_R from both Eqs. (2.5), (2.21) we find the relation (1.1).

2.3 Metastable supersymmetry breaking

The MSB sector is described by the ISS [1] model which is an SQCD with N_f flavours of classically massless quarks and $N_c + 1 \leq N_f < 3N_c/2$. The quarks $\tilde{Q}_{MSB}$, Q_{MSB} generate masses dynamically via the interactions (2.1) with the R-sector as explained above.

Following ISS [1] we introduce canonically normalised fields

$$\Phi_{MSB} = \frac{\tilde{Q}_{MSB} Q_{MSB}}{\Lambda_{MSB}} \ . \tag{2.23}$$

The magnetic description of the gauge theory, then has a classical

$$\mathcal{W}_{\rm cl} = h \, \mathrm{tr}_{N_f} \varphi \Phi_{MSB} \tilde{\varphi} - h \mu_{MSB}^2 \, \mathrm{tr}_{N_f} \Phi_{MSB} \ , \tag{2.24}$$

and dynamical superpotential

$$\mathcal{W}_{\rm dyn} = N \left(h^{N_f} \frac{\det_{N_f} \Phi_{MSB}}{\Lambda_{MSB}^{N_f - 3N}} \right)^{\frac{1}{N}} \ , \tag{2.25}$$

where $N = N_f - N_c$ and h is a constant. Moreover, $\tilde{\varphi}$ and φ are the magnetic quarks made up from suitable combinations of $\tilde{Q}_{MSB}$ and Q_{MSB}. Using the normalisation (2.23) one easily translates the retrofitted mass term for $\tilde{Q}_{MSB}$ and Q_{MSB}, $m_{Q_{MSB}} \sim \Lambda^3/(16\pi^2 M_p^2)$ into μ_{MSB}^2 as given in Eq. (2.4).

In the metastable vacuum near $\Phi_{MSB} = 0$ supersymmetry is broken by the rank condition at the scale μ_{MSB}. In particular, we have

$$\mathrm{tr}(F_{\Phi_{MSB}^{ij}}) \sim \mu_{MSB} \ . \tag{2.26}$$

This supersymmetry breaking is then gauge mediated to the GUT sector by the messengers $\tilde{f}, f$ and the interaction to Φ_{MSB} arising from the first part of Eq. (2.6). The ususal one-loop diagram with messengers propagating in the loop, generates Majorana mass terms for the gauginos of the GUT-sector,

$$m_\lambda \sim \frac{g^2}{16\pi^2} \frac{\Lambda_{MSB}}{M_p} \frac{\mathrm{tr}(F_{\Phi_{MSB}})}{m_f} \sim \frac{g^2}{16\pi^2} \frac{\Lambda_{MSB}^2}{\Lambda_R} \frac{\mu_{MSSM}}{M_{GUT}} \ . \tag{2.27}$$

Fixing

$$m_\lambda \sim 1 \, \mathrm{TeV} \tag{2.28}$$

we find

$$\Lambda_{MSB} \sim 10^{17} \, \mathrm{GeV}. \tag{2.29}$$

Stability of the MSB sector requires that the messengers are non-tachyonic [16],

$$\frac{\Lambda_{MSB}}{M_p} \mu_{MSB}^2 < m_f^2 = \left(\frac{\Lambda_R}{M_p} M_{GUT} \right)^2 \ , \tag{2.30}$$

and that tunneling to a possible supersymmeric vacuum with $\langle\tilde{f}\rangle, \langle f\rangle \neq 0$ is slow,

$$\frac{\Lambda_R M_{GUT}}{\Lambda_{MSB}} \gg \mu_{MSB} \ . \tag{2.31}$$

Both conditions are fulfilled in our model. Similarly, possible flavor changing effects caused by gravity mediation are small because,

$$m_{3/2} = \frac{\mu_{MSB}^2}{M_p} \lesssim 10^{-2} m_\lambda \ . \tag{2.32}$$

2.4 Doublet-triplet splitting and proton decay

Let us return to the Higgs sector and in particular the Higgs triplets. First we should mention that the main issue with minimal $SU(5)$ GUTs is that they predict too rapid proton decay because of dimension-5 operators generated by terms of the form $QQQL$ or $U^cU^cD^cE^c$ in the effective tree-level superpotential (see [20] for a review). Because of this, even standard minimal $SU(5)$ GUTs require modification. This question is also important for our model as we shall now see.

The Higgs triplets are made heavy by the effective operator

$$\mathcal{W}_3 = \kappa \frac{\Lambda_R}{M_p} H \cdot (tr(\Phi_{GUT}) + \Phi_{GUT}) \cdot \tilde{H} \ , \tag{2.33}$$

where κ represents an unknown constant. The Higgs triplet masses are therefore of order $m_{H_3} \sim \kappa\Lambda_R M_{GUT}/M_p \sim \kappa M_{GUT}^3/M_p^2$. Note that the effective mass is proportional to $\mathrm{tr}(\Phi_{GUT})+\Phi_{GUT} = 2\,\mathrm{diag}(1, 1, 1, 0, 0)$, so that the combined coupling shares some features with the Dimopoulos-Wilczek form as discussed widely in the context of $SO(10)$ [23–27]. Indeed our model is rather more natural than standard minimal $SU(5)$ for precisely the same reason as $SO(10)$, namely because the meson field M is not traceless. We would also argue that the requirement that the two couplings in $\mathcal{W}_3$ be identical could conceivably be met by the underlying physics and is a less distasteful fine-tuning than that which occurs in minimal $SU(5)$. (Note that the renormalization of both couplings is identical in the fully supersymmetric theory.)

Clearly however, if κ is of order unity, then this coupling only partially solves the doublet-triplet mass splitting problem: although dimension-6 operators are sufficiently suppressed due to the large gauge-boson masses, dimension-5 operators are large and proton decay is faster than experimental bounds allow. There is little hope within our model of raising M_{GUT} sufficiently: thus, whereas in the minimal $SU(5)$ one might try to elevate the squark masses of the first and second generation, here, firstly their masses would have to be extremely large to avoid proton decay, secondly the soft SUSY breaking is of the gauge mediated form and hence constrained (and in any case there would still be large 3rd generation contributions and a split-supersymmetry scenario with fine-tuning would be unavoidable [28, 29]).

The simplest and most direct route to slow down proton decay, is to assume that the parameter κ is much larger than 1 in order to compensate the Λ_R/M_p suppression in the coupling $H \cdot (\mathrm{tr}(\Phi_{GUT}) +$

$\Phi_{GUT})\cdot\tilde{H}$, and the naive estimate is altered by unknown physics. At the level of the superpotential this does not happen. Additional terms mixing Φ_{GUT} and $H\tilde{H}$ in the Kahler potential could be significant if they are suppressed by a parameter much smaller than inverse powers of Λ_R. This could in principle occur perturbatively if there were states coupling Φ_{GUT} and $H, \tilde{H}$ that had masses lighter than Λ_R, or possibly nonperturbatively if the H and $\tilde{H}$ are themselves composite fields of $SU(N_c)$.

The mileage in the previous idea is probably limited however, since, at best, one recovers the minimal supersymmetric $SU(5)$ which is already known to have difficulty accommodating sufficiently heavy Higgs triplets to avoid proton decay whilst at the same time maintaining gauge unification [28, 29]. If one is willing to go to product "GUT" groups, such as Pati-Salam models, or models based on flipped $SU(5)$, then the doublet-triplet mass splitting problem can be easily avoided. In the latter case for example, the GUT symmetry is broken by VEVs of a $\mathbf{10}$ and $\bar{\mathbf{10}}$ rather than an adjoint Higgs, and the doublet and triplet masses are automatically split. Unfortunately in this case one would have to abandon the adjoint of $SU(N_f)$ which arose rather nicely from the confinement of $SU(N_c)$. In addition it is unclear how $\mathbf{10}$'s and $\bar{\mathbf{10}}$'s would interact (if at all) in the ADS superpotential of the confining R-sector.

Given the similarity of the coupling to the Dimopoulos-Wilczek solution to the doublet-triplet problem, a natural avenue to explore in this class of models is embedding the $SU(5)$ structure within $SO(10)$ which we leave for future study. The reader is referred to [20] for further references to the doublet-triplet mass-splitting problem.

3. Discussion

We have presented an extremely compact formulation of a supersymmetric Grand Unified $SU(5)$ theory. Our model has the following features:

Supersymmetry is broken spontaneously by a long-lived metastable vacuum state of a hidden MSB sector. This supersymmetry breaking is communicated to the GUT theory via gauge mediation and generates gaugino masses which can be made $\sim 10^2 \div 10^3$ GeV. Squark, slepton and higgsino mass splittings follow from this in the standard gauge mediation way.

The model is fully natural with all mass-parameters generated dynamically via the retrofitted couplings to the gluino condensate of the R-sector. In particular, by choosing the dynamical scale of the R-sector to be $\Lambda_R \sim 10^{14}$ GeV, we generate the μ-parameter of the Standard Model, $\mu_{MSSM} \sim 10^2 \div 10^3$ GeV, which in turn generates the required electro-weak symmetry breaking scale $\sim 10^2$ GeV.

Remarkably, the GUT scale $M_{GUT} \sim 10^{16.5}$ GeV $\gg \mu_{MSSM}$ is also dynamically generated in our model. This follows from the fact that the adjoint Higgs required in the GUT sector is identified with the traceless part of the meson matrix of the R-sector. The GUT $SU(5)$ group arises from gauging the $SU(5)$ flavour group of the R-sector, and we show that the required spontaneous breaking of

$SU(5) \to SU(3) \times SU(2) \times U(1)$ does occur at the scale $M_{GUT} \sim \sqrt{\Lambda_R M_p} \sim 10^{16.5}$ GeV.

Hence we have presented a simple and natural (modulo proton decay) model of susy GUT which can explain the values of the symmetry-breaking scales and their hierarchies. The model is weakly coupled and fully calculable including the soft-susy breaking terms.

Acknowledgements

We thank Sakis Dedes and Stefan Förste for useful discussions.

A. Bounds on additional Planck suppressed operators

In Sect. 2 we have made a certain selection among the Planck suppressed operators. In this appendix we will look at the interactions up to $1/M_p^2$ that we have neglected so far.

Let us start with the operators that involve gauge fields as well as matter fields as in (2.1). In Eq. (2.1) we have neglected three types of operators,

$$\Delta\mathcal{W}_1 = \frac{\lambda_1}{16\pi^2 M_p^2}\left[\mathrm{tr}\left(W_R^2\right)\mathrm{tr}(\tilde{Q}_\mathrm{R} Q_\mathrm{R})\right], \tag{A.1}$$

$$\Delta\mathcal{W}_2 = \frac{\lambda_2}{16\pi^2 M_p^2}\left[\mathrm{tr}\left(W_{MSB}^2\right)\mathrm{tr}(\tilde{X} X)\right], \tag{A.2}$$

$$\Delta\mathcal{W}_3 = \frac{\lambda_3}{16\pi^2 M_p^2}\left[\mathrm{tr}\left(W_{GUT}^2\right)\mathrm{tr}(\tilde{X} X)\right], \tag{A.3}$$

where the X is symbolic for all possible matter fields $Q_\mathrm{R}, Q_{MSB}, f, H$. $\Delta\mathcal{W}_2$ and $\Delta\mathcal{W}_3$ are harmless because $\mathrm{tr}(W_{MSB}^2)$ and $\mathrm{tr}(W_{GUT}^2)$ do not acquire significant vacuum expectation values. $\Delta\mathcal{W}_1$ gives a mass of the order of $\Lambda_\mathrm{R}^3/(16\pi^2 M_p^2) \sim \mu_\mathrm{MSSM}$ to the Q_R fields. However, this term has to be compared to the second term of (2.11) which also appears in the F-term for the GUT-field. Inserting the vacuum expectation value for $\tilde{Q}_\mathrm{R} Q_\mathrm{R} \sim \Lambda_\mathrm{R} M_{GUT}$ we find that $\Delta\mathcal{W}_1$ is suppressed by a factor of $\Lambda_\mathrm{R}^{1/2}/(16\pi^2 M_p^{1/2}) \lesssim 10^{-4}$ compared to $\mathcal{W}_\mathrm{R}$. Therefore $\Delta\mathcal{W}_1$ is harmless as well. Overall,

$$\lambda_1, \lambda_2, \lambda_3 \text{ can be } \mathcal{O}(1). \tag{A.4}$$

The second class of possible additional operators involves four matter fields as in (2.6) or (2.10) and is suppressed by one power of $1/M_p$. We have the following possibilities,

$$\Delta\mathcal{W}_4 = \frac{\lambda_4}{M_p}\left[\mathrm{tr}(\tilde{Q}_{\mathrm{R}}Q_{\mathrm{R}})\,\mathrm{tr}(\tilde{Q}_{MSB}Q_{MSB})\right], \tag{A.5}$$

$$\Delta\mathcal{W}_5 = \frac{\lambda_5}{M_p}\left[\mathrm{tr}\left([\tilde{Q}_{MSB}Q_{MSB}]^2\right) + c_5\left[\mathrm{tr}(\tilde{Q}_{MSB}Q_{MSB})\right]^2\right], \tag{A.6}$$

$$\Delta\mathcal{W}_6 = \frac{\lambda_6}{M_p}\left[\mathrm{tr}(\tilde{Q}_{MSB}Q_{MSB})\,\mathrm{tr}(\tilde{H}H)\right], \tag{A.7}$$

$$\Delta\mathcal{W}_7 = \frac{\lambda_7}{M_p}\left[\mathrm{tr}\left([\tilde{H}H]^2\right) + c_7\left[\mathrm{tr}(\tilde{H}H)\right]^2\right], \tag{A.8}$$

$$\Delta\mathcal{W}_8 = \frac{\lambda_8}{M_p}\left[\mathrm{tr}(\tilde{H}H\tilde{f}f) + c_8\,\mathrm{tr}(\tilde{H}H)\,\mathrm{tr}(\tilde{f}f)\right], \tag{A.9}$$

$$\Delta\mathcal{W}_9 = \frac{\lambda_9}{M_p}\left[\mathrm{tr}\left([\tilde{f}f]^2\right) + c_9\left[\mathrm{tr}(\tilde{f}f)\right]^2\right]. \tag{A.10}$$

Inserting the VEV $\langle\tilde{Q}_{\mathrm{R}}Q_{\mathrm{R}}\rangle \sim \Lambda_{\mathrm{R}}M_{GUT}$ we find that $\Delta\mathcal{W}_4$ gives an additional contribution,

$$\Delta\mu^2_{MSB} = \lambda_4\frac{\Lambda_{\mathrm{R}}}{M_p}M_{GUT}\Lambda_{MSB} = \lambda_4\,16\pi^2\left(\frac{M_p}{\Lambda_{\mathrm{R}}}\right)^{\frac{3}{2}}\mu^2_{MSB} \gtrsim 10^9\lambda_4\mu^2_{MSB}. \tag{A.11}$$

To keep our MSB scale at the desired[2] value we therefore have to require,

$$\lambda_4 \lesssim 10^{-9}. \tag{A.12}$$

Interactions of the type $\Delta\mathcal{W}_5$ have two undesirable effects since they lead to linear terms in the potential through $F_{\Phi_{MSB}} = \mu^2_{MSB} + \lambda_5(\Lambda_L^2/M_p)\,\Phi_{MSB} + \ldots$. This can either directly destabilise the metastable minimum or cause a shift in the messenger mass M_f that, in turn again destabilizes the SUSY breaking vacuum. This leads to the constraint [16],

$$\frac{\lambda_5\Lambda^2_{MSB}}{M_p} \lesssim \min\left[0.1\,\mu_{\mathrm{MSB}}, 10^{-2}\frac{\tilde{\lambda}_{\mathrm{f}}\Lambda_{\mathrm{R}}\mathrm{M}_{\mathrm{GUT}}\mathrm{M}_{\mathrm{p}}}{\lambda_{\mathrm{f}}\Lambda^2_{\mathrm{MSB}}}\right], \tag{A.13}$$

where λ_f and $\tilde{\lambda}_f$ are the constants of order one in front of the second and third term in Eq. (2.6). The first part of Eq. (A.13) is the more constraining and leads to

$$\lambda_5 \lesssim 10^{-7}. \tag{A.14}$$

An interaction of type $\Delta\mathcal{W}_6$ would turn the Higgs fields into messengers. At first this looks like a very nice feature. Unfortunately, it also leads to a very large mass term for the Higgs field. This mass comes again from the contribution to the $F_{\Phi_{MSB}} = \mu^2_{MSB} + \lambda_6(\Lambda_{\mathrm{R}}/M_p)\tilde{H}H + \ldots$. The cross terms lead to a contribution of

$$\Delta m^2_H = 2\lambda_6\mu^2_{MSB}\frac{\Lambda_{\mathrm{R}}}{M_p}. \tag{A.15}$$

[2] One might consider the possibility that $\Delta\mathcal{W}_4$ gives indeed the dominant contribution to μ_{MSB}. However, it turns out that the Landau pole of the MSB-sector, Λ_{MSB}, is then too close to μ_{MSB}.

For the Higgs doublet that is part of H and $\tilde{H}$ the mass must be of the order of the electroweak scale and we need

$$\lambda_6 \lesssim 10^{-11}. \tag{A.16}$$

Neither $\tilde{H}, H$ nor $\tilde{f}, f$ aquire any significant (bigger than the electroweak scale) expectation values. Therefore the remaining interactions $\Delta\mathcal{W}_7$,$\Delta\mathcal{W}_8$ and $\Delta\mathcal{W}_9$ provide only additional Planck mass suppressed higher order interactions between the Higgses and the messengers. These interactions are not very constrained and

$$\lambda_7, \lambda_8, \lambda_9 \text{ can be } \mathcal{O}(1). \tag{A.17}$$

Overall the discussion of this appendix shows that the interactions $\Delta\mathcal{W}_4$,$\Delta\mathcal{W}_5$ and $\Delta\mathcal{W}_6$ should be highly suppressed or, preferably, prevented by some mechanism of the underlying theory. All other terms can appear with their natural coefficients of order one.

References

[1] K. Intriligator, N. Seiberg and D. Shih, "Dynamical SUSY breaking in meta-stable vacua," JHEP **0604** (2006) 021 hep-th/0602239.

[2] S. A. Abel, C. S. Chu, J. Jaeckel and V. V. Khoze, "SUSY breaking by a metastable ground state: Why the early universe preferred the non-supersymmetric vacuum," JHEP **0701**, 089 (2007) [arXiv:hep-th/0610334].

[3] N. J. Craig, P. J. Fox and J. G. Wacker, "Reheating metastable O'Raifeartaigh models," arXiv:hep-th/0611006.

[4] W. Fischler, V. Kaplunovsky, C. Krishnan, L. Mannelli and M. Torres, "Meta-stable supersymmetry breaking in a cooling universe," arXiv:hep-th/0611018.

[5] S. A. Abel, J. Jaeckel and V. V. Khoze, "Why the early universe preferred the non-supersymmetric vacuum. II," JHEP **0701**, 015 (2007) [arXiv:hep-th/0611130].

[6] L. Anguelova, R. Ricci and S. Thomas, "Metastable SUSY breaking and supergravity at finite temperature," arXiv:hep-th/0702168.

[7] A. E. Nelson and N. Seiberg, "R symmetry breaking versus supersymmetry breaking," Nucl. Phys. B **416**, 46 (1994) [arXiv:hep-ph/9309299].

[8] J. R. Ellis, C. H. Llewellyn Smith and G. G. Ross, "Will The Universe Become Supersymmetric?," Phys. Lett. B **114** (1982) 227.

[9] S. Dimopoulos, G. R. Dvali, R. Rattazzi and G. F. Giudice, "Dynamical soft terms with unbroken supersymmetry," Nucl. Phys. B **510** (1998) 12 [arXiv:hep-ph/9705307].

[10] M. A. Luty, "Simple gauge-mediated models with local minima," Phys. Lett. B **414** (1997) 71 [arXiv:hep-ph/9706554].

[11] M. Dine, J. L. Feng and E. Silverstein, "Retrofitting O'Raifeartaigh models with dynamical scales," Phys. Rev. D **74**, 095012 (2006) [arXiv:hep-th/0608159].

[12] M. Dine and J. Mason, "Gauge mediation in metastable vacua," arXiv:hep-ph/0611312.

[13] R. Kitano, H. Ooguri and Y. Ookouchi, "Direct mediation of meta-stable supersymmetry breaking," arXiv:hep-ph/0612139.

[14] C. Csaki, Y. Shirman and J. Terning, "A simple model of low-scale direct gauge mediation," arXiv:hep-ph/0612241.

[15] S. A. Abel and V. V. Khoze, "Metastable SUSY breaking within the standard model," arXiv:hep-ph/0701069.

[16] H. Murayama and Y. Nomura, "Gauge mediation simplified," arXiv:hep-ph/0612186.

[17] O. Aharony and N. Seiberg, "Naturalized and simplified gauge mediation," arXiv:hep-ph/0612308.

[18] H. Murayama and Y. Nomura, "Simple scheme for gauge mediation," arXiv:hep-ph/0701231.

[19] H. P. Nilles, "Phenomenological aspects of supersymmetry," arXiv:hep-ph/9511313.

[20] S. Raby, "Grand unified theories," arXiv:hep-ph/0608183.

[21] S. Dimopoulos and H. Georgi, "Softly Broken Supersymmetry And SU(5)," Nucl. Phys. B **193** (1981) 150. N. Sakai, "Naturalness In Supersymmetric Guts," Z. Phys. C **11** (1981) 153.

[22] I. Affleck, M. Dine and N. Seiberg, "Dynamical Supersymmetry Breaking In Supersymmetric QCD," Nucl. Phys. B **241** (1984) 493.

[23] S. Dimopoulos and F. Wilczek, "Incomplete Multiplets In Supersymmetric Unified Models," NSF-ITP-82-07, unpublished; "Supersymmetric Unified Models," *In *Erice 1981, Proceedings, The Unity Of The Fundamental Interactions*, 237-249*

[24] K. S. Babu and S. M. Barr, "Natural suppression of Higgsino mediated proton decay in supersymmetric SO(10)," Phys. Rev. D **48**, 5354 (1993) [arXiv:hep-ph/9306242].

[25] K. S. Babu and S. M. Barr, "Eliminating the d = 5 proton decay operators from SUSY GUTs," Phys. Rev. D **65**, 095009 (2002) [arXiv:hep-ph/0201130].

[26] R. Kitano and N. Okada, "Dynamical doublet-triplet Higgs mass splitting," Phys. Rev. D **64**, 055010 (2001) [arXiv:hep-ph/0105220].

[27] R. Kitano, "Dynamical GUT breaking and mu-term driven supersymmetry breaking," Phys. Rev. D **74**, 115002 (2006) [arXiv:hep-ph/0606129].

[28] T. Goto and T. Nihei, "Effect of RRRR dimension 5 operator on the proton decay in the minimal SU(5) SUGRA GUT model," Phys. Rev. D **59**, 115009 (1999) [arXiv:hep-ph/9808255].

[29] H. Murayama and A. Pierce, "Not even decoupling can save minimal supersymmetric SU(5)," Phys. Rev. D **65**, 055009 (2002) [arXiv:hep-ph/0108104].